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Ground Ring of α-Symmetries and Sequence of RNS String Theories

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Abstract

We construct a sequence of nilpotent BRST charges in RNS superstring theory, based on new gauge symmetries on the worldsheet, found in this paper. These new local gauge symmetries originate from the global nonlinear space-time α-symmetries, shown to form a noncommutative ground ring in this work. The important subalgebra of these symmetries is $U(3) \times X_6$, where X_6 is solvable Lie algebra consisting of 6 elements with commutators reminiscent of the Virasoro type. We argue that the new BRST charges found in this work describe the kinetic terms in string field theories around curved backgrounds of the $AdS \times CP_n$-type, determined by the geometries of hidden extra dimensions induced by the global α-generators. The identification of these backgrounds is however left for the work in progress.

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Introduction

In our recent works [1], [2], [3] we have shown that RNS superstring theory is invariant under the set of global non-linear space-time symmetries (α-symmetries) that mix matter and ghost degrees of freedom and enlarge the space-time symmetry group pointing at their relation to hidden space-time dimensions. The generators of these symmetries (α-generators) can be classified in terms of superconformal ghost cohomologies H_n [3], with each ghost cohomology class contributing its associate hidden dimension to the theory [3]. Typically, the α-generators of H_n $(n > 0)$ have the form:

$$L_n \sim \oint \frac{dz}{2i\pi} e^{n\phi} F_{\frac{n^2}{2}+n+1}(z) \tag{1}$$

in the positive ghost number representation (where ϕ is the bosonized superconformal ghost field, and $F_{\frac{n^2}{2}+n+1}(z)$ are primary matter fields of dimension $\frac{n^2}{2} + n + 1$ composed of RNS bosons and fermions X's and ψ's) or

$$L_n \sim \oint \frac{dz}{2i\pi} e^{-(n+2)\phi} F_{\frac{n^2}{2}+n+1}(z) \tag{2}$$

in the negative ghost cohomology representation $H_{-n-2}(n > 0)$ [3]. The bosonic and fermionic ghost fields β, γ, b and c are bosonized as usual according to [4]

$$\gamma(z) = e^{\phi-\chi}(z); \beta(z) = e^{\chi-\phi}\partial\chi(z) \equiv \partial\xi e^{-\phi}(z)$$

and

$$c(z) = e^{\sigma}(z); b = e^{-\sigma}(z)$$

The important property of the α-symmetries is that there exist no analogues of global space-time transformations induced by the operators (1) at ghost numbers below n (the analogues at pictures above n can be obtained with the help of the picture-changing). At the same time, the α-transformations induced by the generators (2) at the negative ghost numbers have no analogues at pictures above $-n-2$; the analogues at pictures $-n-3$ and below can be obtained with the help of the inverse picture-changing.

From now on, for the rest of the paper we shall concentrate on operators of the type (1) with positive ghost numbers. The α-generators with positive ghost numbers in the form (1) are typically not BRST-invariant (they don't commute with supercurrent terms of the BRST charge) and are therefore incomplete. That is, while they do generate the space-time symmetries, these symmetries are incomplete - for example, the operators (1)

do not act on the $b-c$-ghost sector of the theory, while the full version α-symmetries have to include the $b-c$ ghosts as well. Such a situation is similar to the well-known elementary example of the BRST non-invariant Lorenz rotation generator in the RNS formalism

$$L^{mn} = \oint \frac{dz}{2i\pi} \psi^m \psi^n(z) \tag{3}$$

where ψ's are the RNS fermions: although this symmetry operator does generate the Lorenz rotations for the fermions and fully satisfies the commutation relations of the rotation group, it does not act on the RNS bosons (which of course must also be symmetric under the space-time rotations), and such an incompleteness is directly related to the non-invariance of this operator. Therefore, in order to restore the BRST-invariance of the α-generators (1) (as well as that of the elementary generator (3) of space-time rotations) one needs to add the correction terms that also ensure (perhaps up to picture-changing) that the improved BRST-invariant symmetry generators are complete, inducing the underlying global symmetries for all the relevant fields in the theory. The correction terms can be obtained by the so called K-operator procedure which is defined as follows. Let $L = \oint \frac{dz}{2i\pi} V(z)$ be some global symmetry generator, incomplete (in the sense described above) and not BRST-invariant, satisfying

$$[Q_{brst}, V(z)] = \partial U(z) + W(z) \tag{4}$$

and therefore

$$[Q_{brst}, L] = \oint \frac{dz}{2i\pi} W(z) \tag{5}$$

where V and W are some operators of conformal dimension 1 and U is some operator of dimension zero. Introduce the dimension 0 K-operator:

$$K(z) = -4ce^{2\chi - 2\phi}(z) \equiv \xi \Gamma^{-1}(z) \tag{6}$$

satisfying

$$\{Q_{brst}, K\} = 1 \tag{7}$$

where $\xi = e^{\chi}$ and $\Gamma^{-1} = 4c\partial\xi e^{-2\phi}$ is the inverse picture-changing operator. Suppose that the K-operator (6) has a non-singular OPE with $W(z)$:

$$K(z_1)W(z_2) \sim (z_1 - z_2)^N Y(z_2) + O((z_1 - z_2)^{N+1}) \tag{8}$$

where $N \geq 0$ and Y is some operator of dimension $N+1$. Then the complete BRST-invariant symmetry generator $\tilde{L}$ can be obtained from the incomplete non-invariant symmetry generator L by the following transformation:

$$\begin{aligned} L \to \tilde{L}(w) = L + \frac{(-1)^N}{N!} \oint \frac{dz}{2i\pi}(z-w)^N : K\partial^N W : (z) \\ + \frac{1}{N!} \oint \frac{dz}{2i\pi} \partial_z^{N+1}[(z-w)^N K(z)] K\{Q_{brst}, U\} \end{aligned} \tag{9}$$

where w is some arbitrary point on the worldsheet. It is straightforward to check the invariance of $\tilde{L}$ by using some partial integration along with the relation (7) as well as the obvious identity

$$\{Q_{brst}, W(z)\} = -\partial(\{Q_{brst}, U(z)\}) \tag{10}$$

that follows directly from (4). The corrected invariant $\tilde{L}$-generators are then typically of the form

$$\tilde{L}(w) = \oint \frac{dz}{2i\pi}(z-w)^N \tilde{V}_{N+1}(z) \tag{11}$$

(see the rest of the paper for the concrete examples) with the conformal dimension $N+1$ operator $\tilde{V}_{N+1}(z)$ in the integrand satisfying

$$[Q_{brst}, \tilde{V}_{N+1}(z)] = \partial^{N+1}\tilde{U}_0(z) \tag{12}$$

where $\tilde{U}_0$ is some operator of dimension zero. One unusual property of the invariant $\tilde{L}$-operators is that, while being the generators of global symmetries in space-time, they also appear to depend on an arbitrary point w on a worldsheet, except for the case $N=0$ (the latter case is particularly represented by the space-time rotation generator as will be shown below). It is easy to see, however, that such a w-dependence is not in contradiction with the global properties of the $\tilde{L}$-generators as it will not appear in any correlation functions. Indeed, since all the w derivatives of $\tilde{L}(w)$ are BRST exact:

$$\begin{aligned} \partial_w^k \tilde{L}(w) = \{Q_{brst}, \partial_w^{k-1} b_{-1} \tilde{L}(w)\} \\ k = 1, ..., N \end{aligned} \tag{13}$$

where b is the b-ghost field and $b_{-1} = \oint \frac{du}{2i\pi} b(u)$, changing the worldsheet location of $\tilde{L}$ leaves it invariant up to BRST-trivial terms, that do not contribute to correlators. For this reason, it is clear that the full OPE of any two $\tilde{L}(w)$'s is non-singular and all the OPE terms, except for the contributions of the order of $(w_1 - w_2)^0$, are BRST exact:

$$\tilde{L}_I(w_1)\tilde{L}_J(w_2) = C_{IJ}^K \tilde{L}_K(w_2) + [Q_{brst}, ...] \tag{14}$$

Therefore the $\tilde{L}$-operators form the ground ring with C_{IJ}^{K} being the structure constants of the ring (see the rest of the paper for concrete examples). In this paper we shall particularly concentrate on the properties of the $BRST$-exact derivatives of the $\tilde{L}$-operators. These BRST-exact operators are of interest to us as they turn out to generate new *local* gauge symmetries on the worldsheet (as opposed to global space-time symmetries induced by the $\tilde{L}$-operators themselves). In particular, consider the first derivative of any $\tilde{L}$-operator: $\partial\tilde{L} = \{Q_{brst}, b_{-1}\tilde{L}\}$. This integral has conformal dimension 1 (accordingly the integrand has overall conformal dimension 2) and turns out to generate local gauge symmetries in RNS theory (demonstrated below in this paper). Structurally, it is similar to the well-known conformal symmetries generated by the integral of the dimension 2 stress-energy tensor $\oint T(z)$ which is also BRST-exact, given by the commutator of Q_{brst} with the integral of the b-ghost. As it is well-known, all the derivatives of the stress-energy tensor T in conformal field theory are the symmetry generators as well, giving rise to infinite conformal symmetry in two dimensions. Similarly, just as the first derivatives of $\tilde{L}$'s are the local symmetry generators, the higher order derivatives of $\tilde{L}(w)$ in w induce local symmetries as well. The difference, however, is that in this case one can have only the finite number (up to N) of non-vanishing derivatives of $\tilde{L}$ (13). So the gauge symmetries induced by the derivatives of the $\tilde{L}$-operators are in fact finite-dimensional, although, as will be demonstrated below, they do appear to have Virasoro-like structure, as their commutators involve solvable subalgebras that look somewhat like a "truncated" Virasoro algebra. Given the BRST-exact generators of local symmetries T_I (where I labels the generators) it is straightforward to construct (using the analogy with the stress-tensor) the analogues B_I of the b-ghost that is in the adjoint representation of the local gauge symmetry group. Then, if one is able to find analogues C_I of the c-ghost which must be canonical conjugates of B_I, it is straightforward to construct the nilpotent BRST operator associated with these gauge symmetries. By definition, the BRST operator is given by [5]

$$Q = \sum_{I} C^I T_I + \frac{1}{2}\sum_{I,J,K} f^{IJK} C_I C_J B_K \tag{15}$$

where f^{IJK} are the structure constants of the gauge symmetry group. This operator is nilpotent, provided that the B_I-ghosts are in the adjoint of the gauge symmetry group and f^{IJK} satisfy the Jacobi identities. In this paper, using the chain of the local gauge symmetries inherited from the ground ring of the global α-generators, we shall construct a

sequence of nilpotent BRST charges that can be classified in terms of the ghost cohomologies (corresponding to the underlying α-symmetries) We conjecture that these new BRST operators (while technically constructed of the currents in the CFT of flat-dimensional superstring theory) describe RNS string theories in various *curved* backgrounds (particularly defining the kinetic terms of string field theories built around the curved backgrounds), although we leave the analysis of the cohomologies of these operators for the future work which is currently in progress [6]. The rest of the paper is organized as follows. In the section 2 we review the structure of α-symmetry generators constructed in our previous works [2], [3] and show them to form a ground ring. The derivatives of the α-generators are BRST-exact; they induce local gauge symmetries mixing matter and ghost sectors of RNS superstring theory and can be represented as BRST commutators with certain non-minimal ghost fields. In the section 3 we derive the generalized B_I and C_I ghosts associated with these new gauge symmetries for the superconformal ghost number 1 case and construct the nilpotent BRST charge in the first non-trivial ghost cohomology H_1. In the section 4 we extend our construction to higher ghost number generators, deriving the associate $B-C$ ghost systems leading to nilpotent BRST charges related to gauge symmetries derived from α-generators at minimal ghost numbers 2 nad 3. In the concluding section we discuss possible implications of our results, particularly in relation to the problem of building string field theories around curved backgrounds.

1a. Ghost cohomologies: review of some definitions

Before we start, we shall briefly review some definitions related to ghost cohomologies (that are used to classify the α-symmetries), with some useful modifications (as compared to previous papers [2], [3]) The α-symmetry generators (1),(2) in the positive and negative ghost picture representations have been often referred to as the elements of positive and negative ghost cohomologies respectively. By definition, the negative ghost cohomologies $H_{-n}(n \geq 3)$ consist of physical (BRST-invariant and non-trivial) existing at minimal negative picture $-n$ and the pictures below $(-n-1, -n-2, ...)$, but not above $-n$. The pictures $-n$ and below are related by the usual picture-changing procedure; however, a picture-changing transformation applied to such an operator at the minimal negative picture $-n$ annihilates it, so there is no version of the elements of H_{-n} at superconformal ghost pictures above $-n$. Example of elements of H_{-n} are the α-generators (2) at pictire $-n$. Note that n generally takes the values of -3 and below, as the cohomologies H_{-1} and H_{-2} are empty [3]. As for explicit examples, at this point we only have been able to

identify the elements of H_{-n} up to $n = 5$; for H_{-n} at $n > 5$ the expressions for the operators become quite cumbersome and so far we have not been able to find them explicitly; nevertheless at this time there is no evident reason to exclude the existence of H_{-n} for $n > 5$. Next, the *positive* ghost cohomologies H_n (where $n > 0$) consist of physical (BRST-invariant and nontrivial) operators existing at minimal *positive* picture n and above. The pictures n and above are related by the usual picture-changing procedure; however, an inverse picture-changing transformation applied to such an operator at the minimal positive picture n annihilates it, so there is no version of the elements of H_{-n} at superconformal ghost pictures below n. The isomorphic positive and negative picture representations (1) and (2) for the α-generators hint that there is an similarity between the elements of H_n and H_{-n-2}, conjectured in our previous works [2], [3]. Things, however, appear to be a bit more subtle. The negative picture representation (2) of the α-generators is BRST-invariant and the symmetries generated by them are complete (up to picture changing). The positive picture expressions (1) for the α-symmetry generators are not, on the other hand, BRST-invariant and are not complete (in the sense described above). So they are not, strictly speaking, the elements of $H_n (n > 0)$, even though there exist no versions of the incomplete α-transformations below picture n. In order to make them BRST-invariant , one has to add the correction terms, using the K-operator procedure (9). The *complete* BRST-invariant expressions (9) for the positive picture n α-generators can be, on the contrary, transformed by the inverse picture changing operations to lower pictures (as will be demonstrated below), although the ghost-matter mixing always persists and one is never able to decouple the ghost and the matter degrees of freedom for such operators, even at superconformal picture zero. So strictly speaking, it is not quite accurate to identify the operators (1),(9) with the elements of positive ghost cohomologies. Nevertheless for the sake of brevity and convenience below in the text we shall sometimes refer to operators, existing at minimal positive picture n and above but not below n (such as the "abbreviated" α-generators (1)), as the "elements of H_n" (in characters) even if such operators are not BRST-invariant. At the same time, the complete positive picture α-generators (9) can be described rigorously and more adequately in terms of the $b-c$ ghost cohomologies R_{2n} that will be defined in the next section.

2. Ground Ring of α-Generators and Associate Local Gauge Symmetries

In string theory the global space-time symmetries are typically generated by primary fields of conformal dimension 1 (commuting with BRST charge but not BRST-trivial), while local gauge symmetries are induced by BRST exact operators (that can have various

conformal dimensions and are not necessarily primary) , given by commutators of BRST operator with appropriate ghost fields in the adjoint representation of the gauge group. For example, the generators of local gauge symmetries on the worldsheet, the stress-energy tensor T and the supercurrent S, are the dimension 2 and $\frac{3}{2}$ fields given by

$$T = \{Q_0, b\} \tag{16}$$

and

$$S = [Q_0, \beta], \tag{17}$$

while the dimension 1 primaries

$$L^m = \oint \frac{dz}{2i\pi} \partial X^m \tag{18}$$

and

$$L^{mn} = \oint \frac{dz}{2i\pi} \psi^m \psi^n \tag{19}$$

generate Lorenz translations and rotations on the worldsheet. Here

$$Q_0 = \oint \frac{dz}{2i\pi} (cT - bc\partial c - \frac{1}{2}\gamma\psi_m \partial X^m - \frac{1}{4} b\gamma^2)$$

is the standard BRST operator in RNS theory, having overall superconformal picture zero; we have marked it with the 0 subscript, as below we shall encounter the sequence of alternative nilpotent BRST operators $Q_n (n > 0)$, existing at minimal positive superconformal pictures other than zero (that appear to imitate RNS superstring theories at various curved backgrounds). Before we start discussing the α-symmetries and the associate local gauge symmetries, let us consider (as a warm up example) the elementary case of the space-time rotational symmetry in RNS theory. As was noted above, the space-time rotation operator L^{mn} is not BRST-invariant and, as a consequence it is incomplete as it generates rotational symmetry transformations only for ψ's but not for X's. To construct a complete version of the generator of Lorenz rotations, which acts both on X's and ψ's one needs to improve L^{mn} with bc-ghost dependent terms using the K-operator prescription. In case of the rotation operator (19) the K-operator prescription (9) gives $N = 0$ and the complete BRST-invariant operator for the space-time rotations is

$$L^{mn} \to \tilde{L}^{mn} = L^{mn} - 2 \oint \frac{dz}{2i\pi} c\xi e^{-\phi} \partial X^{[m} \psi^{n]} \tag{20}$$

It is then straightworward to check that, up to a picture-changing transformation, $\tilde{L}^{mn}$ generate the Lorenz rotations for both ψ's and ∂X's, e.g.

$$\Gamma[\epsilon^{mn}\tilde{L}_{mn}\partial X^p] = \Gamma\epsilon_{pn}\partial X^n + [Q_0, ...] \tag{21}$$

Note that the BRST-invariant rotation operator $\tilde{L}^{mn}$ is outside the small Hilbert space, as it manifestly depends on ξ. The next, far less trivial example of global space-time supersymmetries in superstring theory is given by the hierarchy of α-symmetries. These global space-time symmetries are realised non-linearly, mixing the matter and the ghost sectors of RNS superstring theory and can be classified in terms of ghost cohomologies. In our previous works [2], [3] we have analyzed the properties of these unusual symmetries, showing them to originate from hidden extra dimensions of space-time, with each particular ghost cohomology subsector of α-symmetries adding up an extra dimension to space-time symmetry group. The α-symmetry also turns out to play an important role in the correspondence between strings and QCD dynamics, as vertex operators for the octet of gluons with field-theoretic single pole structure of the scattering amplitudes, can be constructed by α-transforming a standard photon vertex operator with SU(3) subgroup of α-generators, which truncated versions are characterized by the lowest three minimal superconformal ghost numbers $1, 2$ and 3 (given the minimal ghost number - extra dimension correspondence, each of 3 hidden dimensions can be associated with particular colour-anticolour pair in this approach [3]). The first and the simplest example of α-symmetry is given by the transformations which truncated generator is characterized by the minimal superconformal ghost number 1. It can be checked that the full matter+ghost RNS action:

$$\begin{aligned} S_{RNS} &= S_{matter} + S_{bc} + S_{\beta\gamma} \\ S_{matter} &= \frac{1}{2\pi}\int d^2z(\partial X_m\bar{\partial}X^m + \psi_m\bar{\partial}\psi^m + \bar{\psi}_m\partial\bar{\psi}^m) \\ S_{bc} &= \frac{1}{2\pi}\int d^2z(b\bar{\partial}c + \bar{b}\partial\bar{c}) \\ S_{\beta\gamma} &= \frac{1}{2\pi}\int d^2z(\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma}) \end{aligned} \tag{22}$$

is invariant under the following transformations (with α being a global parameter):

$$\begin{aligned} \delta X^m &= \alpha\{2e^{\phi}\partial\psi^m + \partial(e^{\phi}\psi^m)\} \\ \delta\psi^m &= -\alpha\{e^{\phi}\partial^2X^m + 2\partial(e^{\phi}\partial X^m)\} \\ \delta\gamma &= \alpha e^{2\phi-\chi}(\psi_m\partial^2X^m - 2\partial\psi_m\partial X^m) \\ \delta\beta &= \delta b = \delta c = 0 \end{aligned} \tag{23}$$

so that

$$\begin{aligned}\delta S_{matter} = -\delta S_{\beta\gamma} = \frac{1}{2\pi}\int d^2z(\bar{\partial}e^{\phi})(\psi_m\partial^2X^m - 2\partial\psi_m\partial X^m)\\ \delta S_{bc} = \delta S_{RNS} = 0\end{aligned} \tag{24}$$

The generator of the transformations (23) is given by

$$L_{\alpha+} = \oint\frac{dz}{2i\pi}e^{\phi}(\psi_m\partial^2X^m - 2\partial\psi_m\partial X^m) \equiv \oint\frac{dz}{2i\pi}e^{\phi}F(X,\psi) \tag{25}$$

where it is convenient to introduce the notation for the dimension $\frac{5}{2}$ primary field:

$$F(X,\psi) = \psi_m\partial^2X^m - 2\partial\psi_m\partial X^m \tag{26}$$

As in the case of the rotation generator, the integrand of the α -generator (25) is a primary field of dimension 1, however it is not BRST-invariant since it doesn't commute with the supercurrent terms of the BRST charge; so similarly one has to introduce the bc-dependent correction terms to make it BRST-invariant, using the K-operator prescription (9).

The BRST-invariant extension of $L_{\alpha+}$ is constructed by using the K-operator prescription (9) requiring (as it is easy to check) $N = 2$. The commutator of BRST charge Q_0 with the integrand of $L_{\alpha+}$ is straightforward to compute. In addition to the dimension $\frac{5}{2}$ primary field $F(X,\psi)$ introduced above, it is also convenient to introduce the dimension 2 primary field

$$L(X,\psi) = 2\partial\psi_m\psi^m - \partial X_m\partial X^m \tag{27}$$

Along with the matter stress tensor $T_{matter} = -\frac{1}{2}\partial X_m\partial X^m - \frac{1}{2}\partial\psi_m\psi^m$ and the matter supercurrent $G = -\frac{1}{2}\psi_m\partial X^m$ the L and F matter primaries satisfy the following useful OPEs:

$$\begin{aligned}G(z)L(w) &= \frac{F(w)}{z-w} + ...\\ G(z)F(w) &= \frac{L(z)}{(z-w)^2} + \frac{1}{4}\frac{\partial L(w)}{z-w} + ...\\ F(z)L(w) &= -\frac{24G(w)}{(z-w)^3} + \frac{4\partial G(w) - 2F(w)}{(z-w)^2} + \frac{4\partial^2G(w) + 4\partial F(w) - 6\partial^2\psi_m\partial X^m}{z-w} + ...\\ F(z)F(w) &= -\frac{22d}{(z-w)^5} + \frac{2L(w) + 20T_{matter}(w)}{(z-w)^3} + \frac{\partial L(w) + 10\partial T_{matter}(w)}{(z-w)^2}\\ &+\frac{1}{z-w}(3\partial^3\psi_m\psi^m(w) - 6\partial^2\psi_m\partial\psi^m(w) + 6\partial^3X_m\partial X^m + \partial^2X_m\partial^2X^m)(w) + ...\\ L(z)L(w) &= \frac{6d}{(z-w)^4} + \frac{1}{(z-w)^2}(16T_{matter}(w) - 4L(w))\\ &+... + \frac{1}{(z-w)}(8\partial T_{matter}(w) - 2\partial L(w))\end{aligned} \tag{28}$$

(where d is the number of space-time dimensions) Using the OPE's (28), the commutator of Q_0 with the integrand of $L_{\alpha+}$ gives

$$\{Q_0, e^{\phi}F(X(z),\psi(z))\} = \partial(ce^{\phi}F) + e^{2\phi-\chi}(FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial LP^{(1)}_{\phi-\chi}) + be^{3\phi-2\chi}FP^{(1)}_{2\phi-2\chi-\sigma} \tag{29}$$

where the conformal weight n polynomials $P^{(n)}_f(\phi_1(z),...,\phi_n(z))$ are the generalized Hermite polynomials defined as

$$P^{(n)}_f(\phi_1(z),...,\phi_n(z)) = e^{-f(\phi_1(z),...,\phi_n(z))}\frac{\partial^n}{\partial z}e^{f(\phi_1(z),...,\phi_n(z))} \tag{30}$$

with f being some function of the fields $\phi_1(z),,\phi_n(z)$ (note that exponents in the definition are multiplied algebraically rather than in terms of OPE). For example, for $P^{(1)}_{\phi-\chi}$ one has $f(\phi,\chi) = \phi - \chi$ and therefore $P^{(1)}_{\phi-\chi} = \partial\phi - \partial\chi$. So we have to consider the K-operator prescription (9) with $U = ce^{\phi}F$, $W = e^{2\phi-\chi}(FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial LP^{(1)}_{\phi-\chi}) + be^{3\phi-2\chi}FP^{(1)}_{2\phi-2\chi-\sigma}$ and, accordingly, $N = 2$. The straightforward evaluation of the OPE between K and $\partial^2 W$ gives:

$$\begin{aligned}
:K\partial^2(e^{2\phi-\chi}(FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial LP^{(1)}_{\phi-\chi})) :&= -8c\xi((FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial LP^{(1)}_{\phi-\chi})) \\
-\frac{1}{4} : L\partial^2(be^{3\phi-2\chi}FP^{(1)}_{2\phi-2\chi-\sigma}) :&= \partial^2(e^{\phi}F) - e^{\phi}FP^{(2)}_{2\phi-2\chi-\sigma} \\
&\quad -8c\xi(FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial LP^{(1)}_{\phi-\chi})
\end{aligned} \tag{31}$$

so the full BRST-invariant extension of the α-generator L_α is given by

$$\begin{aligned}
\tilde{L}_\alpha(w) = \oint \frac{dz}{2i\pi}(z-w)^2[\frac{1}{2}e^{\phi}FP^{(2)}_{2\phi-2\chi-\sigma}(z) + \\
4c\xi(FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial LP^{(1)}_{\phi-\chi}) - 24\partial cce^{2\chi-\phi}F]
\end{aligned} \tag{32}$$

Note that, unlike the case of the rotation generator ($N = 0$ case), the full BRST-invariant expression $\tilde{L}_{\alpha+}(w)$ for the α-generator, obtained by the K-operator transformation (9) with $N = 2$, depends on an arbitrary worldsheet point w. This is related to the fact that $\tilde{L}_{\alpha+}$ is an element of the $b-c$ ghost cohomology R_2, described below. Namely, the $b-c$ ghost cohomologies R_N are defined as follows. Recall that, while a $\beta-\gamma$ picture for an RNS operator refers to its superconformal ghost number, a $b-c$-picture N [7] for some operator L refers to its representation in the form $L = \oint\frac{dz}{2i\pi}(z-w)^N V(z)$ with V being

a dimension $N+1$ operator satisfying $[Q_0, V] = \partial^{N+1}U$ where U is some operator of dimension 0, so L is invariant and w is some arbitrary point on a worldsheet. For example the $N=0$ case corresponds to standard integrated vertex operators in open string theory and $N=-1$ case gives an unintegrated dimension 0 vertex operator located at w. Just as $\beta-\gamma$ pictures can be changed by using the picture-changing operator

$$\Gamma(w) =: \delta(\beta)G(z) :=: e^{\phi}G(w) := -\frac{1}{2}e^{\phi}\psi_m\partial X^m - \frac{1}{4}be^{2\phi-\chi}P^{(1)}_{2\phi-2\chi-\sigma} + c\partial\xi \tag{33}$$

where G is the full matter+ghost worldsheet supercurrent, the operator changing the $b-c$ pictures can be obtained by the bosonic moduli integration in the functional integral for RNS amplitudes and is given by [7]

$$Z(w) =: b\delta(T)(w) = \oint \frac{dz}{2i\pi}(z-w)^3[bT(z) + 4c\partial\xi\xi e^{-2\phi}T^2(z)] \tag{34}$$

.

Just as $\beta-\gamma$ ghost cohomology H_N consists of physical operators that exist at minimal $\beta-\gamma$ picture N and above, but cannot be represented at pictures below N by using picture-changing transformation with Γ or its inverse, the $b-c$ ghost cohomologies $R_N (N > 0)$ consist of physical operators that exist at minimal $b-c$ picture N or above, but cannot be related to pictures below N through any transformation by Z. Thus the complete generator (20) of space-time rortations is the element of R_0, while the complete α-generator $\tilde{L}_{\alpha+}$ is the element of R_2. In the following sections we will also encounter the examples of α-generators of R_4 and R_6, along with their associate gauge symmetries on the worldsheet leading to appearance of new BRST charges.

3. New Local Gauge Symmetries and New BRST Charges

It is straightforward to show that the full BRST-invariant operator $\tilde{L}_{\alpha+}$ (32) induces the full set of global space-time symmetries for the matter and the ghost variables in RNS formalism (including the $b-c$ sector), unlike the "abbreviated" operator $L_{\alpha+}$ (25) generating the truncated version (23) of the α-symmetry. One seemingly unusual property of $\tilde{L}_{\alpha+}(w)$ is that, while generating global space-time symmetry, it explicitly depends on an arbitrary worldsheet coordinate w. This ambiguity, however, can be easily resolved if we note that all of the non-vanishing derivatives of $\tilde{L}_{\alpha+}$ in w (that is, $\partial\tilde{L}_{\alpha+}(w)$ and $\partial^2\tilde{L}_{\alpha+}(w)$) are BRST-exact, therefore there is no dependence on w up to BRST-trivial

terms (e.g. an insertion of $\tilde{L}_{\alpha+}$ in any correlator won't depend on w) It is not difficult to see that the relevant BRST commutators are given by:

$$
\begin{aligned}
L^1_{\alpha+} \equiv \partial\tilde{L}_{\alpha+}(w) = \{Q_0, B_{\alpha+}(w)\} \equiv \{Q_0, b_{-1}\tilde{L}_\alpha(w)\} \\
L^2_\alpha \equiv \partial^2\tilde{L}_{\alpha+}(w) = \{Q_0, \partial B_{\alpha+}(w)\}
\end{aligned}
\tag{35}
$$

It is also helpful to have a manifest expression for $B_{\alpha+}(w)$. Simple calculation gives

$$
\begin{aligned}
B_{\alpha+}(w) = \oint\frac{dz}{2i\pi}(z-w)^2\{-2be^{\phi}FP^{(1)}_{2\phi-2\chi-\sigma}(z) \\
+8\xi(FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial LP^{(1)}_{\phi-\chi}) + 24\partial ce^{2\chi-\phi}F\}
\end{aligned}
\tag{36}
$$

Next, it is straightforward to check that the dimension 1 BRST-exact operator $L^1_{\alpha+}$ (which integrand has conformal dimension 2) generates *local* transformations for RNS fields that mix the ghost and the matter sectors leaving S_{RNS} invariant:

$$
\begin{aligned}
\delta(\partial X^m(w)) = \epsilon(w)\{-16\partial(c\xi G)\psi^m - 8c\xi F\psi^m \\
+8c\xi P^{(2)}_{\phi-\chi}\partial X^m - 4\partial(c\xi {P^{(1)}}_{\phi-\chi}\partial X^m) + 2c\xi {P^{(1)}}_{\phi-\chi}\partial^2 X^m \\
+2\partial((e^{\phi}P^{(2)}_{2\phi-2\chi-\sigma} - 24\partial cc\partial\xi\xi e^{-\phi})\psi^m)\} \\
+\partial\epsilon(w)\{-16c\xi G\psi^m - 4c\xi {P^{(1)}}_{\phi-\chi}\partial X^m \\
+2(e^{\phi}P^{(2)}_{2\phi-2\chi-\sigma} - 24\partial cc\partial\xi\xi e^{-\phi})\psi^m\} \\
\delta\psi^m = \epsilon(w)\{-2(e^{\phi}P^{(2)}_{2\phi-2\chi-\sigma} - 24\partial cc\partial\xi\xi e^{-\phi})\partial X^m + 16c\xi G\partial X^m \\
+8c\xi {P^{(2)}}_{\phi-\chi}\psi^m - 4\partial(c\xi P^{(1)}_{\phi-\chi}\psi^m)\} - 4\partial\epsilon(w)c\xi P^{(1)}_{\phi-\chi}\psi^m \\
\delta c(w) = \epsilon(w)ce^{\phi}F(w) \\
\delta b(w) = -24\epsilon(w)c\partial\xi\xi e^{-\phi}F(w) \\
\delta\beta(w) = \epsilon(w)(4\xi F + 8c\partial\xi\xi e^{-\phi}L(w)) \\
\delta\gamma = 0
\end{aligned}
\tag{37}
$$

It is important to note that the symmetry transformations (37) are induced by the generator $L^1_{\alpha+}(w)$ applied to RNS variables located at the *same* point w. If RNS variables located at w are transformed by $L^1_{\alpha+}$ located at any point other than w, S_{RNS} is not symmetric under such transformations.

Next, it is a bit tedious but straightforward to check that the ring of the gauge symmetry generators $L^1_{\alpha+}$ and $L^2_{\alpha+}$ is commutative, i.e.

$$
[L^1_{\alpha+}(w), L^2_{\alpha+}(w)] = 0 \tag{38}
$$

and in addition $L_{\alpha+}^{(1,2)}(w)$ commute with the ghost variable $B_{\alpha+}(w)$, as well as with all its non-vanishing derivatives (that is, the first and the second derivatives in w). Note that, while the dimension 1 generator $L_{\alpha+}^1$ inducing local gauge transformations at ghost number 1-level can be regarded as a counterpart of conformal transformations induced by $T_{-1} = \oint T(z)$ at ghost number zero, the commutation relations (38) indicate that the dimension 1 ghost variable $B_{\alpha+}$, associated with the gauge symmetries (37) is the prototype of the integral of the usual b-ghost $\oint b$, associated with the conformal transformations. It is important to stress that that neither $L_{\alpha+}^1$ nor $B_{\alpha+}$ are related to $\oint T$ and $\oint b$ by any picture-changing (just as the global α-symmetry transformations (23) have no picture zero analogue). The gauge symmetry (37) is essentially different from conformal symmetry and $L_{\alpha+}^1$ is a generator physically different from $\oint T$. Having the gauge symmetry (37), its generator $L_{\alpha+}^1$ and the associate ghost $B_{\alpha+}$ (analogue of $\oint b$, the commutative ring of $L_{\alpha}^{1,2}$ (which elements also commute with $B_{\alpha+}$ and its derivatives) the only missing component to construct a nilpotent BRST charge at the H_1-level, related to the gauge symmetry (37), is the analogue $C_{\alpha+}$ of the c-ghost, conjugate to $B_{\alpha+}$. Since b and c ghosts satisfy the canonical relation

$$\{\oint b, c\} = 1, \tag{39}$$

the $B_{\alpha+}$ and $C_{\alpha+}$ ghosts have to satisfy

$$\{B_{\alpha+}, C_{\alpha+}\} = 1 \tag{40}$$

The object that is able to satisfy the anticommutator (40) with $B_{\alpha+}$ must be a *local* field of conformal dimension -1. Since the usual c-ghost is also a primary field, our goal now is to construct the conformal dimension -1 primary field satisfying (40). Since the expression (36) for $B_{\alpha+}$ is at picture 1, $C_{\alpha+}$ satisfying (40) must be at picture -1. However, simple structural and dimensional analysis shows that there exist no picture -1-object with such properties. For this reason, we shall replace the canonical relation (40) with

$$\{\Gamma^{-2} B_{\alpha+}, C_{\alpha+}\} = \Gamma \tag{41}$$

with $\Gamma = \{Q_0, \xi\}$ being the direct picture-changing operator or, equivalently, the picture-transformed unit operator and $\Gamma^{-2} =: \Gamma^{-1}\Gamma^{-1}$: being the square of the inverse picture-changing $\Gamma^{-1} = 4c\partial\xi e^{-2\phi}$ satisfying : $\Gamma\Gamma^{-1} := 1$.

Note that, even though $B_{\alpha+}$ is off-shell, picture-changing transformations (both the direct and the inverse) are still well-defined for it: since the $L^1_{\alpha+}$ is on-shell by construction, it can be picture-transformed in a well-defined manner; then, since Γ^{-2} is invariant, by definition

$$\Gamma^{-2}L^1_{\alpha+} = \{Q_0, B^{(-2)}_{\alpha+}\} \equiv \{Q_0, \Gamma^{-2}B_{\alpha+}\} \tag{42}$$

Applying the inverse picture-changing twice with Γ^{-1} it is straightforward to obtain the picture-transformed expression for $B_{\alpha+}$ at ghost number -1:

$$\begin{aligned}
B^{(-1)}_{\alpha+}(w) \equiv \Gamma^{-2}B_{\alpha+}(w) = \oint \frac{dz}{2i\pi}(z-w)^2\{-8\partial cce^{3\phi-4\chi} \\
\times\{\frac{1}{2}P^{(2)}_{-\sigma}[-\frac{3}{8}\partial^2 L + \frac{1}{4}\partial LP^{(1)}_{-16\phi+3\chi-3\sigma} \\
+L\times(-\frac{3}{2}\partial^2\phi + \frac{11}{8}\partial^2\chi + \frac{3}{8}\partial^2\sigma - 4(\partial\phi)^2 + \frac{5}{8}(\partial\chi)^2 \\
+\frac{1}{8}(\partial\sigma)^2 + 6\partial\phi\partial\chi - \frac{1}{2}\partial\phi\partial\sigma + \frac{7}{4}\partial\chi\partial\sigma)] \\
+\frac{1}{6}P^{(3)}_{-\sigma}(-\frac{3}{4}\partial L + L(\frac{1}{4}\partial\sigma - \frac{1}{2}\partial\phi)) + \frac{1}{48}P^{(4)}_{-\sigma}L\} \\
-ce^{2\chi-3\phi}\{P^{(1)}_{-\sigma}\times[-\frac{3}{8}\partial^2 F - \frac{1}{4}\partial FP^{(1)}_{\phi-2\chi+2\sigma} \\
+F[\frac{1}{8}\partial^2\phi + \frac{15}{4}\partial^2\chi - \frac{1}{4}\partial^2\sigma + \frac{13}{8}(\partial\phi)^2 = \\
-3(\partial\chi)^2 - \frac{5}{2}\partial\phi\partial\chi - \frac{3}{2}\partial\phi\partial\sigma + \partial\chi\partial\sigma] \\
+\frac{1}{2}P^{(2)}_{-\sigma}(\frac{-1}{2}\partial F + F(-\frac{3}{2}\partial\phi - \partial\chi)) - \frac{1}{24}P^{(3)}_{-\sigma}\}(z)
\end{aligned} \tag{43}$$

Next, it is straightforward to check that the conformal dimension -1 primary field $C_{\alpha+}$, satisfying the picture-transformed canonical relation (40) with $B_{\alpha+}$ is given by:

$$\begin{aligned}
C_{\alpha+}(w) = \frac{1}{2}e^{3\phi-\chi}\{F(\frac{1}{3}P^{(3)}_{\phi-\chi} + \frac{1}{2}\partial\phi P^{(2)}_{\phi-\chi}) + GL(\frac{1}{2}P^{(2)}_{\phi-\chi} + \partial\phi P^{(1)}_{\phi-\chi} + \frac{1}{2}\partial FP^{(2)}_{\phi-\chi}) \\
+\partial GLP^{(1)}_{2\phi-\chi} + G\partial LP^{(1)}_{\phi-\chi} + \frac{1}{2}\partial^2 GL + \partial G\partial L\} \\
+be^{4\phi-2\chi}\{\frac{1}{2}GFP^{(1)}_{\phi-\chi-\frac{3}{4}\sigma}P^{(1)}_{\phi-\chi} + \frac{1}{12}LP^{(3)}_{\phi-\chi}P^{(1)}_{\phi-\chi-\frac{3}{4}\sigma} + \frac{1}{16}\partial LP^{(2)}_{\phi-\chi}P^{(1)}_{\phi-\chi-\frac{3}{4}\sigma}\} \\
+\partial bbe^{5\phi-3\chi}\{-\frac{1}{8}P^{(1)}_{\phi-\chi-\frac{3}{4}\sigma}P^{(2)}_{2\phi-2\chi-\sigma} + \frac{1}{32}P^{(3)}_{2\phi-2\chi-\sigma}\}
\end{aligned} \tag{44}$$

The expression (44) for $C_{\alpha+}$ is at the "picture cohomology H_2" i.e. it has no analogues at pictures below 2 - in particular, that's the reason why one has to transform $B_{\alpha+}$ in

order to satisfy the canonical relation (41). Having the gauge symmetry generators $L^{1,2}_{\alpha+}$, the associate $B_{\alpha+}$ and $C_{\alpha+}$-ghosts, as well as the commutation relations (38), it is now straightforward to construct the nilpotent BRST-charge in the H_1 ghost cohomology which, by definition, is given by

$$Q_1 = {C_{\alpha+}}^1 L^1_\alpha + {C_{\alpha+}}^2 L^2_\alpha \tag{45}$$

where

$$\begin{aligned} {C_{\alpha+}}^1 &\equiv C_\alpha+, \\ {C_{\alpha+}}^2 &= \oint \frac{dz}{2i\pi} C_{\alpha+}(z) \end{aligned} \tag{46}$$

To construct the manifest expression for Q_1 in terms of an integral of BRST current, it is convenient transform the full BRST-invariant expression for $\tilde{L}_{\alpha+}(w)$ (as well as those for $L^1_{\alpha+} = \partial\tilde{L}_{\alpha+}(w)$ and $L^2_{\alpha+} = \partial^2\tilde{L}_{\alpha+}(w)$) to -1 picture, as $C_{\alpha+}$ only exists at picture 2 and above, and Q_1 should have minimal superconformal picture 1. Applying Γ^{-2} to $\tilde{L}_{\alpha+}(w)$ (32) twice, it is straightforward to obtain

$$\begin{aligned} \tilde{L}^{(-1)}_{\alpha+}(w) = \oint \frac{dz}{2i\pi}(z-w)^2\{-8\partial^2 c\partial cce^{3\chi-4\phi}[-\frac{3}{8}\partial^2 L \\ +\partial L(-4\partial\phi + 3\partial\chi - \frac{3}{4}\partial\sigma) \\ +L(-\frac{3}{2}\partial^2\phi + \frac{11}{8}\partial^2\chi + \frac{3}{8}\partial^2\sigma - 4(\partial\phi)^2 + \frac{5}{8}(\partial\chi)^2 \\ +\frac{1}{8}(\partial\sigma)^2 + 6\partial\phi\partial\chi - \frac{1}{2}\partial\phi\partial\sigma + \frac{7}{4}\partial\chi\partial\sigma)] \\ -\partial cce^{2\chi-3\phi}[-\frac{3}{8}\partial^2 F + \partial F(-\frac{1}{4}\partial\phi + \frac{1}{2}\partial\chi - \frac{1}{2}\partial\sigma) \\ +F(\frac{1}{8}\partial^2\phi + \frac{15}{4}\partial^2\chi - \frac{1}{4}\partial^2\sigma + \frac{13}{8}(\partial\phi)^2 \\ -3(\partial\chi)^2 - \frac{5}{2}\partial\phi\partial\chi - \frac{3}{2}\partial\phi\partial\sigma + \partial\chi\partial\sigma)]\} \end{aligned} \tag{47}$$

Substituting (44) and (47) into the BRST charge (45) and evaluating the relevant OPE's we obtain, upon a straightforward although somewhat lengthy computation, the remarkably simple expression:

$$Q_1 = \oint \frac{dz}{2i\pi}\{ce^{\phi}FP^{(1)}_{\phi-\chi} - \frac{1}{8}e^{2\phi-\chi}(LP^{(2)}_{2\phi-2\chi-\sigma} + 2GF) - \partial cc\xi L\}(z) \tag{48}$$

This BRST charge is nilpotent by construction and its nilpotence can also be checked by direct computation. It is an on-shell operator commuting with Q_0 and is an element of H_1 ghost cohomology, i.e. it is unrelated to the standard BRST charge Q_0 by any

picture-changing transformation. As was stressed above, the charge Q_1 (48) is an independent BRST charge, originating from the ground ring of α-symmetries, with independent cohomology of physical states.

4. α-Generators of Higher Ghost Cohomologies: a Review

For uncompactified critical RNS theory, Q_1 of H_1 is the only additional BRST charge present, as for this case there exist only one global α-symmetry with two associate generators of local gauge symmetries deduced from $\tilde{L_{\alpha+}}$. In non-critical or compactified cases, however, there are additional α-generators, at minimal positive ghost numbers $n > 1$ [3], due to interactions with the Liouville mode or the compactified dimensions. For completeness, below we shall give a short review of the basic properties of the α-generators at higher minimal ghost numbers, briefly summarizing the results of [3].

In general, for a d-dimensional RNS theory, there exist $d+1$ additional α-symmetries in $b-c$ ghost cohomology R_2 (with their truncated non-invariant versions (1) having minimal superconformal ghost number 1 prior to adding the correction terms). For the brevity, below we shall give the truncated expressions for these operators (i.e. before adding the $b-c$ ghost correction terms); the K-operator procedure is performed for these operators quite similarly to the $L_{\alpha+} \to \tilde{L}_{\alpha+}$ case explained above. The remaining $d+1$ truncated α-generators of H_1 (one d-vector and one Lorenz scalar) are given by

$$L_{m\alpha} = \oint \frac{dz}{2i\pi} e^{\phi}\{\partial^2\varphi\psi^m - 2\partial\varphi\partial\psi^m + \partial^2 X^m\lambda - 2\partial X^m\partial\lambda\} \tag{49}$$

and

$$L_{\alpha-} = \oint \frac{dz}{2i\pi} e^{\phi}\{\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda\} \tag{50}$$

where φ and λ are the components of the super Liouville field.

Combined with $\frac{(d+1)(d+2)}{2}$ Poincare symmetries (including the Liouville direction), these $d+2$ ghost-matter mixing symmetries of R_2 enlarge space-time symmetry group from $SO(2,d)$ to $SO(2,d+1)$, bringing in the first extra-dimension

Next, the R_4 cohomology can be shown to contain $(d+3)$ α-symmetries (with their truncated versions having minimal superconformal ghost number 2) which, combined with Poincare symmetries and α-symmetries of H_2 enlarge the space-time symmetry group to

$SO(2,d+2)$, bringing in the second extra-dimension. The truncated expressions for the α-generators of R_4 are given by

$$
\begin{aligned}
L_{\beta+} &= \oint \frac{dz}{2i\pi} e^{2\phi} F_1(X,\psi) F_1(\varphi,\lambda)(z) \\
L_{\beta-} &= -\oint \frac{dz}{2i\pi} e^{2\phi} F_{1m}(X,\lambda) F_1^m(\varphi,\psi)(z) \\
L_{\beta m} &= \oint \frac{dz}{2i\pi} e^{2\phi} (F_1^m(X,\lambda) F_1(\varphi,\lambda) - F_1(X,\psi) F_1^m(\varphi,\psi))(z) \\
L_{\alpha\beta} &= \oint \frac{dz}{2i\pi} e^{2\phi} (\frac{1}{2} F_2(\lambda,\varphi) + L_1(X,\psi)\partial L_1(\varphi,\lambda) - \partial L_1(X,\psi) L_1(\varphi,\lambda))(z)
\end{aligned}
\tag{51}
$$

with the matter+Liouville structures L and F (L_1, F_1 and F_1^m) being the primary fields of dimensions 2 and $\frac{5}{2}$:

$$
\begin{aligned}
F_1(X,\psi) &= \psi_m \partial^2 X^m - 2\partial\psi_m \partial X^m \\
F_1(\varphi,\lambda) &= \lambda\partial^2\varphi - 2\partial\lambda\partial\varphi \\
F_1^m(X,\lambda) &= \lambda\partial^2 X^m - 2\partial\lambda\partial X^m \\
F_1^m(\varphi,\psi) &= \psi^m\partial^2\varphi - 2\partial\psi^m\partial\varphi \\
L_1(X,\psi) &= \partial X_m \partial X^m - 2\partial\psi_m\psi^m \\
L_1(\varphi,\lambda) &= (\partial\varphi)^2 - 2\partial\lambda\lambda
\end{aligned}
\tag{52}
$$

and $F_2(\lambda,\varphi)$ being the primary field of dimension 5:

$$
\begin{aligned}
F_2(\varphi,\lambda) &= \frac{1}{4}(\partial\varphi)^5 - \frac{3}{4}\partial\varphi(\partial^2\varphi)^2 + \frac{1}{4}(\partial\varphi)^2\partial^3\varphi \\
&+\lambda\partial\lambda(\partial^3\varphi - (\partial\varphi)^3) - \frac{3}{2}\lambda\partial^2\lambda\partial^2\varphi + 3\partial\lambda\partial^2\lambda\partial\varphi\} \\
&\equiv i : (\oint e^{-i\varphi}\lambda)^3 e^{3i\varphi}\lambda :
\end{aligned}
\tag{53}
$$

Finally, the $d+4$ α-generators of R_6 (with their truncated versions having minimal superconformal ghost number 3) include one Lorenz d-vector and 4 scalars, enlarging the symmetry group to $SO(2,d+3)$, bringing in the third hidden dimension. The truncated

α-generators of R_6 are given by

$$
\begin{aligned}
L_{\gamma+} &= \oint \frac{dz}{2i\pi} e^{3\phi}\{2\partial F_1(X,\psi)F_2(\varphi,\lambda) - F_1(X,\psi)\partial F_2(\varphi,\lambda)\} \\
L_{\gamma m} &= \oint \frac{dz}{2i\pi} e^{3\phi}\{2F_2^m(\psi,\lambda,\varphi)\partial F_1(X,\psi) - \partial F_2(\psi,\lambda,\varphi)F_1(X,\psi) \\
&\quad +2F_2(\varphi,\lambda)\partial F_1^m(X,\lambda) - \partial F_2(\varphi,\lambda)F_1^m(X,\lambda)\} \\
L_{\gamma -} &= \oint \frac{dz}{2i\pi} e^{3\phi}\{2G_2(\psi,\lambda,\varphi)\partial F_1(X,\psi) - \partial G_2(\psi,\lambda,\varphi)F_1(X,\psi) \\
&\quad +3F_{2m}(\psi,\lambda,\varphi)\partial F_1^m(X,\lambda) - 2\partial F_{2m}(\psi,\lambda,\varphi)F_1^m(X,\lambda) - \partial F_2(\lambda,\varphi)F_1(X,\psi)\} \\
L_{\gamma\beta} &= \oint \frac{dz}{2i\pi} e^{3\phi}\{F_3(\varphi,\lambda) + \partial L_1(X,\psi)L_2(\varphi,\lambda) - \frac{4}{11}L_1(X,\psi)\partial L_2(\varphi,\lambda)\} \\
L_{\gamma\alpha} &= \oint \frac{dz}{2i\pi} e^{3\phi} L_{2m}(\varphi,\psi)L_1^m(X,\lambda)
\end{aligned}
\tag{54}
$$

with the additional matter+Liouville blocks given by:

$$
\begin{aligned}
F_2^m(\psi,\lambda,\varphi) &= \partial^2\psi^m\lambda\partial^2\varphi - \psi^m\partial^2\lambda\partial^2\varphi + 3\partial^2\psi^m\partial\lambda\partial\varphi - 3\partial\psi^m\partial^2\lambda\partial\varphi \\
G_2(\psi,\lambda,\varphi) &= 4\partial\psi_m\partial^2\psi^m\partial\varphi - 2\psi_m\partial^3\psi^m\partial\varphi + (2d-4)(\lambda\partial^3\lambda\partial\varphi - 2\partial\lambda\partial^2\lambda\partial\varphi) \\
L_2(\varphi,\lambda) &= -\frac{5}{4}(\partial\varphi)^4\partial\lambda + \frac{3}{4}(\partial^2\varphi)^2\partial\lambda + \frac{3}{2}\partial\varphi\partial^2\varphi\partial^2\lambda - \frac{5}{2}\partial\varphi\partial^3\varphi\partial\lambda \\
&\quad -\frac{1}{4}(\partial\varphi)^2\partial^3\lambda - 4\partial\varphi\partial^2\varphi\partial^2\lambda + \partial^2\varphi\partial^3\varphi\lambda \\
L_2^m(\varphi,\psi) &= -\frac{5}{4}(\partial\varphi)^4\partial\psi^m + \frac{3}{4}(\partial^2\varphi)^2\partial\psi^m + \frac{3}{2}\partial\varphi\partial^2\varphi\partial^2\psi^m - \frac{5}{2}\partial\varphi\partial^3\varphi\partial\psi^m \\
&\quad -\frac{1}{4}(\partial\varphi)^2\partial^3\psi^m - 4\partial\varphi\partial^2\varphi\partial^2\psi^m + \partial^2\varphi\partial^3\varphi\psi^m \\
L_1^m(X,\lambda) &= \partial^2\lambda\psi^m + \lambda\partial^2\psi^m \\
F_3(\varphi,\lambda) &=: (\oint e^{-i\varphi}\lambda)^4 e^{4i\varphi}\lambda :
\end{aligned}
\tag{55}
$$

Although the α-symmetry generators of R_{2n} $b-c$ ghost cohomologies (along with their associate rings of local gauge symmetries) have not yet been constructed explicitly for $n>3$ cases (as the manifest expressions for the α-symmetry generators become extremely cumbersome at higher n's), it seems plausible that the α-symmetries also exist at $n>3$ levels as well, with each subset of the generators from R_{2n} at a given n adding the associate hidden space-time dimension (checked explicitly for $n=1,2,3$ [3]). There are both Lorenz scalars and d-vectors among α-symmetry generators of the first three cohomologies $n\leq 3$; the scalar generators have been shown to form the $SU(3)$ subgroup. If one takes

an open string photon and acts on it with the α-generators of $SU(3)$ subgroup, one obtains an $SU(3)$ octet of vertex operators of gluons which tree level amplitude reproduces that of $SU(3)$ QCD, i.e. this amplitude has a field-theoretic rather than a stringy structure (with only the massless poles present). The absence of an infinite tower of massive poles (typical for standard Veneziano amplitudes) is related to the remarkable property of the α-transformations proven in [3]: if applied to massless states, they produce new physical massless vertex operators; however, the α-transformations applied to any massive intermediate states produce BRST-trivial operators that of course do not contribute to the scattering amplitude. Thus the α-symmetry "erases" the massive poles, to ensure the proper field-theoretic behaviour of the gluon amplitude. Higher order α-generators may also be constructed in critical RNS theory, e.g. by compactifying one of ten dimensions on S^1 and replacing the super Liouville mode with the compactified dimension (and its worldsheet superpartner) in the expressions for the generators of R_2, R_4 and R_6.

5. α-Generators of Higher Ghost Cohomologies: Gauge Symmetries and BRST Charges

Given the truncated versions of the α-generators of R_2, R_4 and R_6, their complete BRST-invariant expressions are straightforward to construct by using the K-operator procedure (9). The construction is totally similar to the case of $\tilde{L}_{\alpha+}$ of R_2, described above. Upon the K-operator transformation, all the truncated α-generators (49), (50) of ghost number 1 become the elements of the $b-c$ ghost cohomology R_2 (similarly to the case of $L_{\alpha+} \to \tilde{L}_{\alpha+}$) considered above). Next, it is straightforward to find out that, upon the K-operator transformation (9) all the truncated α-generators (51) of ghost number 2 become the elements the cohomology R_4, while the truncated operators (54) of ghost number 3 become the elements of R_6. That is, in the case of the truncated superconformal ghost number 2 α-symmetries (51) the K-operator prescription (9) requires $N=4$ and is given by

$$\begin{aligned} L^{(2)} \to \tilde{L}^{(2)}_{R_4}(w) = L^{(2)} - \frac{1}{24}\oint \frac{dz}{2i\pi}(z-w)^4 L\partial^4 W^{(2)}(z) \\ +\frac{1}{24}\oint \frac{dz}{2i\pi}\partial_z^5[(z-w)^4 L]L\{Q_0, U^{(2)}\} \end{aligned} \tag{56}$$

where the operators $W^{(2)}$ and $U^{(2)}$ are defined by the BRST (Q_0) commutator with the integrands of the truncated expressions (51) for the subset of the α- generators $L^{(2)}$ of H_2:

$$\begin{aligned} L^{(2)} &\equiv \oint \frac{dz}{2i\pi} V^{(2)} \\ [Q_0, V^{(2)}] &= \partial U^{(2)} + W^{(2)} \end{aligned} \tag{57}$$

Similarly, for the superconformal ghost number 3 truncated α-symmetries (54) the K-operator prescription requires $N=6$ and is given by

$$L^{(3)} \to \tilde{L}^{(3)}_{R_4}(w) = L^{(2)} - \frac{1}{720}\oint\frac{dz}{2i\pi}(z-w)^6 L\partial^6 W^{(3)}(z) + \frac{1}{720}\oint\frac{dz}{2i\pi}\partial_z^7[(z-w)^6 L]L\{Q_0, U^{(3)}\} \quad (58)$$

where, as previously,

$$L^{(3)} \equiv \oint\frac{dz}{2i\pi}V^{(3)} \qquad [Q_0, V^{(3)}] = \partial U^{(3)} + W^{(3)} \quad (59)$$

Structurally, the complete BRST (Q_0)-invariant α-generators $\tilde{L}^{(k)}(k=1,2,3)$ in the $b-c$ ghost cohomologies R_2, R_4 and R_6 respectively, have the form:

$$\tilde{L}^{(k)}(w) = \oint\frac{dz}{2i\pi}(z-w)^{2k}\tilde{V}^{(2k+1)} \qquad [Q_0, \tilde{V}^{(2k+1)}] = \partial^{2k+1}\tilde{U_k}^{(0)} \quad (60)$$

where $\tilde{V}^{(2k+1)}$ are the integrands of conformal dimension $2k+1$ and $\tilde{U_k}^{(0)}$ are some operators of conformal dimension zero. The ghost-matter structures of the operators $\tilde{L}^{(k)}(k=1,2,3)$ in R_{2k} respectively are given by

$$\begin{aligned}\tilde{L}^{(1)}(w) &= \oint\frac{dz}{2i\pi}(z-w)^2(e^{\phi}R^{(\frac{9}{2})} + ce^{\chi}R^{(4)} + \partial cce^{2\chi-\phi}R^{(\frac{5}{2})})(z)\\ \tilde{L}^{(2)}(w) &= \oint\frac{dz}{2i\pi}(z-w)^4(e^{2\phi}R^{(9)} + ce^{\phi+\chi}R^{(\frac{15}{2})} + \partial cce^{2\chi}R^{(5)})(z)\\ \tilde{L}^{(3)}(w) &= \oint\frac{dz}{2i\pi}(z-w)^6(e^{3\phi}R^{(\frac{29}{2})} + ce^{\chi+2\phi}R^{(12)} + \partial cce^{\phi+2\chi}R^{(\frac{17}{2})})(z)\end{aligned} \quad (61)$$

where $R^{(k)}$ are the operators of conformal dimension k, made of matter fields and various polynomial functions of ghost number currents, such as $P^{(i)}_{\phi-\chi}$ and $P^{(j)}_{2\phi-2\chi-\sigma}$ with various i and j. For the sake of completeness, below we shall give the manifest expressions for some most typical operators $\tilde{L}^{(k)}(w)$ for $k=2$ and 3:

$$\begin{aligned}\tilde{L}_{\beta+}(w) = \oint\frac{dz}{2i\pi}(z-w)^4\{&\frac{1}{24}e^{2\phi}F(X,\psi)F(\varphi,\lambda)P^{(4)}_{2\phi-2\chi-\sigma}\\ &+\frac{1}{6}ce^{\chi+\phi}[\partial GF(X,\psi)F(\varphi,\lambda) + GF(X,\psi)F(\varphi,\lambda)P^{(1)}_{\phi-\chi} + \frac{1}{6}P^{(3)}_{\phi-\chi}(L(X,\psi)F(\varphi,\lambda)\\ &-F(X,\psi)L(\varphi,\lambda)) + \frac{1}{8}P^{(2)}_{\phi-\chi}(\partial L(X,\psi)F(\varphi,\lambda) - F(X,\psi)\partial L(\varphi,\lambda))]\\ &+20\partial cc\partial\xi\xi F(X,\psi)F(\varphi,\lambda)\}\end{aligned} \quad (62)$$

for $k = 2$ and

$$\tilde{L}_{\gamma+} = \frac{1}{720}\oint\frac{dz}{2i\pi}(z-w)^6\{e^{3\phi}(2\partial F_1(X,\psi)F_2(\varphi,\lambda) - F_1(X,\psi)\partial F_2(\varphi,\lambda))P^{(6)}_{2\phi-2\chi-\sigma}\}$$
$$+\oint\frac{dz}{2i\pi}(z-w)^6\{c\xi e^{2\phi}[(2GP^{(2)}_{\phi-\chi} + 4\partial GP^{(1)}_{\phi-\chi} + 2\partial^2 G)$$
$$\times(2\partial F_1(X,\psi)F_2(\varphi,\lambda) - F_1(X,\psi)\partial F_2(\varphi,\lambda))$$
$$+P^{(3)}_{\phi-\chi}(\frac{1}{3}\partial^2 L_1(X,\psi)F_2(\varphi,\lambda) + \frac{2}{3}\partial F_1(X,\psi)\partial L_2(\varphi,\lambda)$$
$$-\frac{1}{6}\partial L_1(X,\psi)\partial F_2(\varphi,\lambda) - \frac{4}{3}\partial F_1(X,\psi)L_2(\varphi,\lambda))$$
$$+P^{(4)}_{\phi-\chi}(\frac{5}{12}\partial L_1(X,\psi)F_2(\varphi,\lambda) + \frac{1}{6}F_1(X,\psi)L_2(\varphi,\lambda)$$
$$-\frac{1}{6}L_1(X,\psi)\partial F_2(\varphi,\lambda)) + \frac{2}{15}P^{(5)}_{\phi-\chi}2L_1(X,\psi)F_2(\varphi,\lambda)]$$
$$+7\partial cc\partial\xi\xi e^{\phi}(2\partial F_1(X,\psi)F_2(\varphi,\lambda) - F_1(X,\psi)\partial F_2(\varphi,\lambda))\} \tag{63}$$

for $k = 3$. Given the expressions (61)-(63), we are now prepared to analyze the symmetry algebra generated by $\tilde{L}^{(k)}$ that originate from R_{2k} global α-symmetries. Just as the simplest global α-generator $\tilde{L}_{\alpha+}(w)$ gives rise to local gauge symmetries, defined by the BRST-exact derivatives $L^1_{\alpha+} = \partial\tilde{L}_{\alpha+}$ and $L^2_{\alpha+} = \partial^2\tilde{L}_{\alpha+}$, the remaining generators of R_2, as well as those of R_4 and R_6 also give rise to their associate gauge symmetries. For the remaining generators of R_2, $\tilde{L}_{\alpha\pm}$ and $\tilde{L}_{\alpha m}$, structure of associate gauge symmetries is simple and similar to that of $\tilde{L}_\alpha$; the generators are given by $L^{(1)}_1 \equiv \partial\tilde{L}^{(1)}, L^{(2)}_2 \equiv \partial^2\tilde{L}^{(1)}$ with $\tilde{L}^{(1)} \equiv (\tilde{L}_{\alpha\pm}, \tilde{L}_{\alpha m})$ All the gauge symmetry generators commute with each other (just as in the case of a single $\tilde{L}_{\alpha+}$ generating commutative ring with two elements $L^\alpha_{1,2}$ considered above). The α-generators $\tilde{L}^{(2),(3)}$ of higher cohomologies R_4 and R_6 give rise to the local gauge symmetries with far more interesting structure. Each of the $d+3$ elements of $\tilde{L}^{(2)}(w) \equiv (\tilde{L}_{\beta\pm}(w), \tilde{L}_{\beta\alpha}(w), \tilde{L}_{\beta m}(w))$ gives rise to 4 BRST exact gauge symmetry generators

$$L^{(2)}_k \equiv \partial^k\tilde{L}^{(2)}(w) = \{Q_0, b_{-1}\partial^{k-1}\tilde{L}^{(2)}\}$$
$$k = 1,2,3,4 \tag{64}$$

while each of the $d+4$ elements of $\tilde{L}^{(3)}(w) \equiv (\tilde{L}_{\gamma\pm}(w), \tilde{L}_{\gamma\beta}, \tilde{L}_{\gamma\alpha}(w), \tilde{L}_{\gamma m}(w))$ gives rise to 6 BRST exact gauge symmetry generators

$$L^{(3)}_k \equiv \partial^k\tilde{L}^{(3)}(w) = \{Q_0, b_{-1}\partial^{k-1}\tilde{L}^{(3)}\}$$
$$k = 1,..,6 \tag{65}$$

To determine the algebra of the gauge symmetry generators $L_k^{(n)}(n = 1, 2, 3)$, we first need to calculate the OPE's of their integrands. We write

$$L_k^{(n)}(w) = \partial_w^k \oint \frac{dz}{2i\pi}(z-w)^{2n} V^{(n)}(z) \tag{66}$$

where the operators $V^{(n)}$ have conformal dimensions $2n+1$ and

$$\begin{aligned} V^{(1)} &\equiv (V_{\alpha\pm}, V_{\alpha m}) \\ V^{(2)} &\equiv (V_{\beta\alpha}, V_{\beta\pm}, V_{\beta m}) \\ V^{(3)} &\equiv (V_{\gamma\beta}, V_{\gamma\alpha}, V_{\gamma\pm}, V_{\gamma m}) \end{aligned} \tag{67}$$

so that $V_{\alpha\pm}$ is the integrand of $\tilde{L}_\pm$ etc. As the manifest expressions for $V^{(n)}$ operators (66), (67) are quite complicated, we shall particularly concentrate on the subgroup of 9 α-generators that are the Lorenz scalars, that is, $(\tilde{L}_{\alpha\pm}, \tilde{L}_{\beta\pm}, \tilde{L}_{\gamma\pm}, \tilde{L}_{\alpha\beta}, \tilde{L}_{\alpha\gamma}, \tilde{L}_{\beta\gamma})$. The OPE calculation is quite cumbersome, although it can be somewhat simplified by using the isomorphism between the operators of R_{2n} (with their truncated versions being at minimal positive picture n) and those of the negative ghost cohomologies H_{-n-2}, explained in [3].

The lengthy computation gives the following table of the operator products:

$$
\begin{aligned}
V_{\gamma\beta}(z_1)V_{\gamma\alpha}(z_2) &= ... + \frac{1}{210}\sum_{k=0}^{6}\frac{(-1)^k k!(6-k)!}{(6+k)!}\frac{\partial^{(k+2)}V_{\alpha\beta}(z_2)^{[6]}}{(z_1-z_2)^{7-k}}\\
V_{\gamma\beta}(z_1)V_{\gamma\pm}(z_2) &= ... + \frac{1}{210}\sum_{k=0}^{6}\frac{(-1)^k k!(6-k)!}{(6+k)!}\frac{\partial^{(k+2)}V_{\beta\pm}(z_2)^{[6]}}{(z_1-z_2)^{7-k}}\\
V_{\gamma\alpha}(z_1)V_{\gamma\pm}(z_2) &= ... + \frac{1}{2520}\sum_{k=0}^{6}\frac{(-1)^k k!(6-k)!}{(6+k)!}\frac{\partial^{(k+4)}V_{\alpha\pm}(z_2)^{[6]}}{(z_1-z_2)^{7-k}}\\
V_{\gamma\beta}(z_1)V_{\beta\pm}(z_2) &= ... + \frac{1}{5}\frac{V^{\gamma\pm}}{(z_1-z_2)^5} + 6\sum_{k=1}^{4}\frac{(-1)^k k!(4-k)!}{(6+k)!}\frac{\partial^{(k+2)}V_{\gamma\pm}(z_2)^{[5]}}{(z_1-z_2)^{5-k}}\\
V_{\gamma\beta}(z_1)V_{\beta\alpha}(z_2) &= ... + \frac{1}{5}\frac{V^{\gamma\alpha}}{(z_1-z_2)^5} + 6\sum_{k=1}^{4}\frac{(-1)^k k!(4-k)!}{(6+k)!}\frac{\partial^{(k+2)}V_{\gamma\alpha}(z_2)^{[5]}}{(z_1-z_2)^{5-k}}\\
V_{\gamma\alpha}(z_1)V_{\beta\alpha}(z_2) &= ... + \frac{1}{5}\frac{V^{[5]}_{\gamma\beta}}{(z_1-z_2)^5} + 6\sum_{k=1}^{4}\frac{(-1)^k k!(4-k)!}{(6+k)!}\frac{\partial^{(k+2)}V_{\gamma\beta}(z_2)^{[5]}}{(z_1-z_2)^{5-k}}\\
V_{\gamma\pm}(z_1)V_{\beta\mp}(z_2) &= ... + \frac{1}{5}\frac{V^{[5]}_{\gamma\beta}}{(z_1-z_2)^5} + 6\sum_{k=1}^{4}\frac{(-1)^k k!(4-k)!}{(6+k)!}\frac{\partial^{(k+2)}V_{\gamma\beta}(z_2)^{[5]}}{(z_1-z_2)^{5-k}}\\
V_{\alpha\pm}(z_1)V_{\gamma\alpha}(z_2) &= ... + \frac{1}{3}\frac{V^{[4]}_{\gamma\pm}}{(z_1-z_2)^3} - \frac{1}{42}\frac{\partial V^{[4]}_{\gamma\pm}}{(z_1-z_2)^2} + \frac{1}{168}\frac{\partial^2 V^{[4]}_{\gamma\pm}}{z_1-z_2}\\
V_{\alpha\pm}(z_1)V_{\gamma\pm}(z_2) &= ... + \frac{1}{3}\frac{V_{\gamma\alpha}}{(z_1-z_2)^3} - \frac{1}{42}\frac{\partial V^{[4]}_{\gamma\alpha}}{(z_1-z_2)^2} + \frac{1}{168}\frac{\partial^2 V^{[4]}_{\gamma\alpha}}{z_1-z_2}\\
V_{\beta\alpha}(z_1)V_{\beta\pm}(z_2) &= ... + \frac{1}{60}\sum_{k=0}^{4}\frac{(-1)^k k!(4-k)!}{(4+k)!}\frac{\partial^{(k+2)}V_{\alpha\pm}(z_2)^{[4]}}{(z_1-z_2)^{5-k}}\\
V_{\beta\alpha}(z_1)V_{\alpha\pm}(z_2) &= ... + \frac{1}{3}\frac{V^{[3]}_{\beta\pm}}{(z_1-z_2)^3} - \frac{1}{30}\frac{\partial V^{[3]}_{\beta\pm}}{(z_1-z_2)^2} + \frac{1}{90}\frac{\partial^2 V^{[3]}_{\beta\pm}}{z_1-z_2}\\
V_{\beta\pm}(z_1)V_{\alpha\mp}(z_2) &= ... + \frac{1}{3}\frac{V^{[3]}_{\beta\alpha}}{(z_1-z_2)^3} - \frac{1}{30}\frac{\partial V^{[3]}_{\beta\alpha}}{(z_1-z_2)^2} + \frac{1}{90}\frac{\partial^2 V^{[3]}_{\beta\alpha}}{z_1-z_2}
\end{aligned}
\tag{68}
$$

where the numbers in the square brackets on the right hand side refer to the superconformal pictures of the generators and we have skipped the OPE terms that are too singular to contribute to the algebra of the α-generators $\tilde{L}^{ij}(w)$ (where the i,j indices stand for α,β or γ) That is, for example, the commutator of $\tilde{L}_{\gamma\beta}(w) = \oint\frac{dz_1}{2i\pi}(z_1-w)^6V_{\gamma\beta}(z_1)$ with $\tilde{L}_{\gamma\alpha}(w) = \oint\frac{dz_2}{2i\pi}(z_2-w)^6V_{\gamma\alpha}(z_2)$ won't be contributed by the terms in the OPE of $V_{\gamma\beta}(z_1)$ and $V_{\gamma\alpha}(z_2)$ with the singularity order of $(z_1-z_2)^{-8}$ or stronger, therefore we skip these terms in the first OPE in the table (68), starting the expansion from the order of $(z_1-z_2)^{-7}$.

Using the OPE table (68) it is not difficult to compute the algebra of the global α-generators and of the local gauge symmetry generators $L_{ij}^m \equiv \partial_w^m \tilde{L}_{ij}$ of the associate gauge symmetries (as before, the m index runs from 1 to $2k$ for the gauge symmetries associated with the generators of R_{2k}; that is, $k = 1$ for $L^m_{\alpha\pm}$, $k = 2$ for $L^m_{\beta\pm}, L^m_{\beta\alpha}$ and $k = 3$ for $L^m_{\gamma\pm}, L^m_{\gamma\alpha}, L^m_{\gamma\beta}$. First of all, simple calculation using the OPE's (68) shows that the 9 global α-generators, $\tilde{L}_{ij} \equiv (\tilde{L}_{\alpha\pm}, \tilde{L}_{\beta\pm}, \tilde{L}_{\gamma\pm}, \tilde{L}_{\beta\alpha}, \tilde{L}_{\gamma\alpha}, \tilde{L}_{\gamma\beta})$ satisfy the commutation relations of $U(3)$:

$$[\tilde{L}_{i_1j_1}, \tilde{L}_{i_2j_2}] = -\delta_{i_1j_2}\tilde{L}_{i_2j_1} + \delta_{i_2j_1}\tilde{L}_{i_1j_2} \tag{69}$$

The computation of commutators of the α-generator's derivaves L^m_{ij} using the table (68) is straightforward as well. It is convenient to redefine the generators $L^m_{ij} \to T^m_{ij}$ according to

$$\begin{aligned} T^m_{\alpha\pm} &= \frac{L^m_{\alpha\pm}}{(2-m)!}(m = 1, 2) \\ T^m_{\beta\alpha} &= \frac{L^m_{\beta\alpha}}{(4-m)!}(m = 1, 2, 3, 4) \\ T^m_{\beta\pm} &= \frac{L^m_{\beta\pm}}{(4-m)!}(m = 1, 2, 3, 4) \\ T^m_{\gamma\alpha} &= \frac{L^m_{\gamma\alpha}}{(6-m)!}(m = 1, ..., 6) \\ T^m_{\gamma\beta} &= \frac{L^m_{\gamma\beta}}{(6-m)!}(m = 1, ..., 6) \\ T^m_{\gamma\pm} &= \frac{L^m_{\gamma\pm}}{(6-m)!}(m = 1, ..., 6) \end{aligned} \tag{70}$$

Then the commutators of T^m_{ij} satisfy

$$[T^m_{i_1j_1}, T^n_{i_2j_2}] = (n-m)(\delta_{i_2j_1}T^{m+n}_{i_1j_2} - \delta_{i_1j_2}T^{m+n}_{i_2j_1}) \tag{71}$$

provided that of $m + n \leq 2k$ where $2k_{r.h.s.}$ is the order of the $R_{2k_{r.h.s.}}$ cohomology of each corresponding operator on the right hand side; Otherwise, in case if $m + n > 2k_{r.h.s.}$, the generators commute. Schematically,

$$\begin{aligned} [T^m_I, T^n_J] = (m-n)f^K_{IJ}T^{m+n}_K(m + n \leq 2k_{r.h.s.}) \\ (m + n \leq 2k_{r.h.s.}) \end{aligned} \tag{72}$$

where the capital indices stand for $I = (i_1, j_1), J = (i_2, j_2)$, etc; f^K_{IJ} are the $U(3)$ structure constants, so the algebra of the gauge symmetries associated to $R_{2k}(k = 1, 2, 3)$

α-generators is isomorphic to $U(3)\times X_6$ where X_6 is solvable Lie algebra consisting of 6 elements $x_1,...,x_6$ with the commutation relations given by

$$\begin{aligned}[x_m,x_n]&=(m-n)x_{n+m}(m,n=1...6;m+n\leq 6)\\ [x_m,x_n]&=0(m+n>6)\end{aligned}\tag{73}$$

Given the T_{ij}^m generators of the $U(3)\times X_6$ gauge symmetries , it isn't difficult to show that the generalized b-ghost fields (in the adjoint of $U(3)\times X_6$) satisfying

$$L_{ij}^m=\{Q_0,\partial^{m-1}\oint B_{ij}\}\tag{74}$$

are given by

$$\oint B_{ij}=b_{-1}\tilde{L}_{ij}\tag{75}$$

similarly to the simplest case (35). Given the generalized b-ghost fields of (74), one can construct the generalized c-ghost fields C_{ij}, conformal dimension -1 primaries and canonical conjugates of B_{ij}, satisfying

$$\{\oint B_{ij},C_{ij}\}=:\Gamma^n:\tag{76}$$

where Γ is again the picture-changing operator with $n=1,2,3$ for $\tilde{L}_{ij}$'s of R_2, R_4 and R_6 respectively. With some effort, it is possible to derive explicit expressions for various C_{ij}'s. For the C_{ij}'s corresponding to $\tilde{L}_{ij}$'s of R_4 the expressions are given by

$$C_{ij}^{(R_4)}=e^{\phi}Ge^{3\phi-\chi}L^{matter}_{ij(\frac{9}{2})}P^{(1)}_{\phi-\frac{10}{3}\chi}+\{\tilde{Q}_0,\partial^2b\partial bbe^{5\phi-2\chi}L^{matter}_{ij(\frac{9}{2})}\}\tag{77}$$

where $F^{matter}_{ij(5)}$ are the conformal dimension 5 matter parts of the $\tilde{L}_{ij}$ operators of R_4 given in (51), $L^{matter}_{ij(\frac{9}{2})}$ are the worldsheet superpartners of $F^{matter}_{ij(5)}$ satisfying

$$\oint\frac{dz}{2i\pi}\{G(z),L^{matter}_{ij(\frac{9}{2})}(w)\}=F^{matter}_{ij(5)}(w)$$

and $\tilde{Q}_0$ is the ghost part of the standard BRST charge:

$$\tilde{Q}_0=Q_0-\oint\frac{dz}{2i\pi}(cT-bc\partial c)\tag{78}$$

The generalized C_{ij}-ghost operators (77) associated with the R_4 gauge symmetry generators (70) are the "elements of H_3" (off-shell operators existing at superconformal ghost

pictures 3 and above, but not below 3) satisfying the canonical relations with the R_4-associated B_{ij}-ghosts, up to the double picture-changing:

$$\{\oint \frac{dz}{2i\pi} B^{(R_4)}_{ij[-1]}, C^{(R_4)}_{ij}\} =: \Gamma^2 : \tag{79}$$

Next, the expressions the generalized c-ghost fields C_{ij}, corresponding to the gauge symmetries associated with the elements (70) of R_6 are given by

$$\begin{aligned} C^{(R_6)}_{ij} &=: e^{\phi} G e^{4\phi-\chi} P^{(1)}_{\phi-3\chi} L^{matter}_{ij(9)} : \\ +\{\tilde{Q}_0, \partial^4 b \partial^3 b \partial^2 b \partial b b e^{7\phi-3\chi} &(\partial P^{(1)}_{\phi-\frac{9}{4}\chi} + P^{(1)}_{\phi-\frac{9}{4}\chi} P^{(1)}_{7\phi-3\chi-7\sigma}) F^{matter}_{ij(\frac{17}{2})}\} \end{aligned} \tag{80}$$

where the conformal dimension $\frac{17}{2}$ primaries $F^{matter}_{ij(\frac{17}{2})}$ are the matter components of the R_6-generators of (54), (61), (63) while the conformal dimension 9 primaries $L^{matter}_{ij(9)}$ are the worldsheet superpartners of $F^{matter}_{ij(\frac{17}{2})}$, satisfying

$$\{\oint \frac{dz}{2i\pi} G(z), F^{matter}_{ij(\frac{17}{2})}(w)\} = L^{matter}_{ij(9)}(w).$$

The generalized C_{ij}-ghost operators (80) associated with the derivatives of the R_6 α-generators (54),(61),(63) are the "elements of H_4" (off-shell operators existing at superconformal ghost numbers 4 and above, but not below 4) satisfying the canonical relations with the R_6-associated B_{ij}-ghosts, up to the triple picture-changing:

$$\{\oint \frac{dz}{2i\pi} B^{(R_6)}_{ij[-1]}, C^{(R_6)}_{ij}\} =: \Gamma^3 : \tag{81}$$

Finally, given the $U(3) \times X_6$ gauge symmetries induced by $T^m_I \equiv T^m_{ij}$ (capital indices stand for the abbreviation of (ij), as previously), the generalized ghosts $B_I \equiv B_{ij}$, $C_I \equiv C_{ij}$ and the symmetry algebra (71), (72) (70), (75), (77) it is straightforward to construct the nilpotent BRST operator associated with the U(3) subgroup of α-symmetries of $R_{2n}(n = 1, 2, 3)$:

$$Q_2 = \sum_I \sum_{m=1}^{6} (C_I)_m T^m_I + \frac{1}{2} \sum_{I,J,K} \sum_{m,n=1}^{m+n \leq 6} (m-n) f^{IJK} (C_I)_m (C_J)_n (B_K)_{-m-n} \tag{82}$$

where f^{IJK} are again the appropriate $U(3)$ structure constants and

$$\begin{aligned} C^n_I &= \oint \frac{dz}{2i\pi} z^{m-2} C_I(z) \\ (B_I)_m &= m! \oint \frac{dw}{2i\pi} \frac{b_{-1} \tilde{L}_K(w)}{w^{m+1}} \end{aligned} \tag{83}$$

Depending on the values of I, J and K of the $U(3)$ indices in (82), the generalized B_I and C_I ghosts entering the expression for Q_2 are related to the various classes of the gauge symmetries derived from the R_2, R_4 or R_6 cohomologies (with the precise expressions given in (43),(44),(74), (77) and (80) for each cohomology case).

While the manifest integral form expression for Q_2 is quite complicated (far more complicated and lengthy than for Q_1 of H_1 constructed in (48)), it can be shown that structurally it consists of three terms which are on-shell (with respect to Q_0) and are the elements of superconformal ghost cohomologies H_1, H_2 and H_3 respectively; while the H_1 part of Q_2 contains Q_1 of (48), the H_2 and H_3 ingrediants are related to the gauge symmetries associated with the geometry α-generators of R_4 and R_6. The BRST operator Q_2 (82) can be shown to commute with the nilpotent BRST charges Q_0 and Q_1 of the lower ghost cohomologies. It appears that Q_2 describes RNS strings in curved background which is shaped by the geometry of the extra dimensions induced by the $U(3)$ subgroup of the α-generators. We leave the detailed analysis of this geometry, as well as the analysis of the cohomologies of Q_1 and Q_2, for the future work. It would be very interesting to generalise the construction described in this paper in order to build the nilpotent BRST charges based on the higher order $b-c$ ghost cohomologies $R_{2n}(n \geq 4)$. The expressions for the α-generators involving the higher ghost numbers are, however, increasingly complicated; understanding their structures clearly requires further effort.

6. Conclusion and Discussion

In this paper we constructed a sequence of new nilpotent BRST charges Q_1 (48) and Q_2 (82) consisting of the currents in superconformal ghost cohomologies H_1, H_2 and H_3. The construction is based on the sequence of hidden local gauge symmetries of the RNS theory, associated with the ground ring of α-generators, that are classified in terms of the $b-c$ ghost cohomologies R_2, R_4 and R_6. The α-generators, in turn, induce global space-time symmetries originating from hidden space-time dimensions. The constructed BRST charges thus appear to describe superstring theories in various curved backgrounds which geometry is shaped by the extra dimensions induced by the global α-symmetries in $R_{2n}(n = 1, 2, 3)$. In principle, the construction of the nilpotent BRST charges charges could be extended to higher $n > 3$, as there is no clear reason to exclude the existence of α-symmetry generators and their ground rings at $b-c$ cohomology levels R_{2n} with $n > 3$. At present, however, we do not have the construction for the $n > 3$ cases, as the expressions for the generators and their OPE's become extremely complicated at $n > 3$ cases, with the technical difficulties always aggravated by the picture-changing. Our preliminary analysis

[6] shows, however, that there exists certain underlying principle defining the structures of the α-generators of higher cohomologies; if one is able to advance in this direction, this could probably simplify the construction of the BRST charges for higher n's. As was noted above, the new BRST charges constructed in this paper define the sequence of RNS string theories constructed in certain curved geometrical backgrounds, typically AdS (in the case of $Q_0 + Q_1$) or $AdS \times CP_n$-type (in the case of linear combination of Q_0, Q_1 and Q_2) [6]; these backgrounds have to be identified by the constraints that the gauge transformations, induced by the ground ring of the α-symmetries, impose on space-time geometry. The work concerning this problem is currently in progress and we hope to clarify the related questions in future works. Since the charges Q_0, Q_1 and Q_2 commute with each other, their combinations , such as $Q_0 + Q_1$, $Q_0 + Q_2$ or $Q_0 + Q_1 + Q_2$ define kinetic terms in certain RNS string field theories (SFT) built around the curved backgrounds, mentioned above. For example, the preliminary analysis of the cohomology of $Q_0 + Q_1$ charge in open string theory shows that its cohomology consists of a single gauge boson:

$$\begin{aligned} V(k) = \oint e^{-3\phi}\{(\vec{A}\partial\vec{X})(\vec{k}\partial\vec{X})(\vec{k}\vec{\psi}) + (\vec{A}\vec{\psi})(\vec{k}\partial\vec{\psi})(\vec{k}\vec{\psi})(\vec{A}\vec{\psi})(\vec{k}\partial\vec{X})^2\}e^{i\vec{k}\vec{X}} \\ \vec{k}\vec{A}(\vec{k}) = 0; (\vec{k})^2 = 0 \end{aligned} \tag{84}$$

with no massive states at all. Such a spectrum is typical for strings in AdS-type background which are known to be dual to gauge theories [8], [9], [10], [11]. In general, we expect the backgrounds, corresponding to new BRST charges found in this paper, to include those of the AdS or $AdS_q \times CP_n$-type [6], with the CP_n-structures related to subgroups of the α-symmetries restricted to certain ghost cohomology classes. The particular background geometries, however, depend on various choices of different subgroups or cosets of the underlying α-symmetries. For example, the $CP_2 \sim \frac{SU(3)}{SU(2)\times U(1)}$ fibration can be obtained by taking the $U(3)$-subgroup (67) of the α-generators, factorizing by $SU(2)$ generated by $\tilde{L}_{\alpha\beta}, \tilde{L}_{\alpha\gamma}$ and $\tilde{L}_{\alpha\beta}$, excluding $\tilde{L}_{\alpha+}$ (playing the role of $U(1)$) and defining certain linear combinations of the remaining α-transformations up to $L_{\alpha-}$ (which is possible since the latter commutes with physical vertex operators [3])

Exploring the new SFT's based on these new BRST charges could be useful in order to develop the non-supersymmetric versions of AdS/CFT. In particular, in the $d = 4$ case the gauge-string duality can be understood as the duality between closed SFT built around the AdS_5 background and the loop equations in $d = 4$ [12] , with the BRST charges of the curved string field theory being dual to the loop operator in non-supersymmetric

QFT. On the other hand, studying the string field theories with BRST charges related to $AdS_q \times CP_n$ structures would be useful to explore some other remarkable examples of AdS/CFT dualities, such as the duality type between IIA strings on $AdS_4 \times CP_3$ and the 't Hooft limit of three-dimensional $SU(N) \times SU(N)$ gauge theory describing the effective worldvolume dynamics of M2 branes [13], [14]. In particular, it would be interesting to relate the symmetries of Bagger-Lambert and ABJM three-dimensional theories to the gauge symmetries associated to appropriate ground rings of α-generators. This, however, would require a better understanding of the R_8 cohomology of operators which structure is still obscure. Another challenging development would be to construct BRST charges that would imitate M-theory dynamics on $AdS_7 \times CP_2$ that could perhaps shed some light on the $M5$-brane worldvolume physics. We hope that the results presented in this paper will be helpful for the progress in these directions.

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